Planck Scale Cosmology and Resummed Quantum Gravity

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We show that, by using amplitude-based resummation techniques for Feynman's formulation of Einstein's theory, we get quantum field theoretic 'first principles' predictions for the UV fixed-point values of the dimensionless gravitational and cosmological constants. Connections to the phenomenological asymptotic safety analysis of Planck scale cosmology by Bonanno and Reuter are discussed.

I. INTRODUCTION

Sometime ago, Weinberg [1] pointed-out that quantum gravity may be asymptotically safe in that the UV behavior of the theory corresponds to a UV-fixed point with a finite dimensional critical surface so that the S-matrix only depends on a finite number of dimensionless parameters. Recently, Bonanno and Reuter [2, 3] have shown, using a realization developed by Reuter [4] of the idea via Wilsonian field space exact renormalization group methods, that one arrives at a purely Planck scale quantum mechanical formulation the inflationary cosmological scenario of Guth and Linde [5, 6] – this is very attractive as it opens the possibility of a deeper understanding of that scenario without the need of the hitherto unseen inflaton scalar field. In what follows, using the new resummed theory [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] of quantum gravity, which is based on Feynman's original approach [17, 18] to the subject, we recover the properties as used in Refs. [2, 3] for the UV fixed point of quantum gravity with the added results that we get 'first principles' predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants in their analysis.

The discussion proceeds as follows. In the next section we review the formulation of Einstein's theory by Feynman, as it is not generally familiar. In Section 3, we present the elements of the resummed version of Feynman's formulation, resummed quantum gravity. Section 4 presents the applications to Planck scale cosmology as it is formulated by Bonanno and Reuter [2, 3]. Section 5 contains our concluding remarks.

II. FEYNMAN'S FORMULATION OF EINSTEIN'S THEORY

In Feynman's approach [17, 18] to quantum gravity, the starting point is that the metric of space-time undergoes quantum field theory fluctuations just like all point-particle fields: we write the metric of space-time as $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the flat Minkowski space background metric and $\kappa = \sqrt{8\pi G_N}$ so that $h_{\mu\nu}(x)$ is the quantum field of the graviton when G_N is Newton's constant. For definiteness and reasons of pedagogy, we specialize the complete theory here, which is

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda) + \sqrt{-g}L^{\mathcal{G}}_{SM}(x) \tag{1}$$

where R is the curvature scalar, g is the determinant of the metric of space-time $g_{\mu\nu}$, Λ is the cosmological constant and $\text{Ł}^{\mathcal{G}}_{SM}(x)$ is the diffeomorphism invariant form of the SM Lagrangian obtained from the well-known SM Lagrangian in Ref. [19] by standard differential-geometric methods [7], to the case of a single scalar field, the Higgs field $\varphi(x)$, with a rest mass set at $m = 120$ GeV [20, 21], in interaction with the graviton so that the relevant Lagrangian is now that already considered by Feynman [17, 18] when ignore the small cosmological constant [22] (we will re-instate it shortly):

$$\begin{aligned}\mathcal{L}(x) &= -\frac{\sqrt{-g}}{2\kappa^2}R + \frac{\sqrt{-g}}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\\ &= \frac{1}{2}\big\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\\ &\quad \bar{h}_{\mu'\sigma,\sigma'}\big\} + \frac{1}{2}\big\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\big\}\\ &\quad -\kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right]\\ &\quad\ -\kappa^2\Big[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\big(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\big)\\ &\quad -2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\Big] + \cdots\end{aligned} \tag{2}$$

where $\varphi_{,\mu} \equiv \partial_\mu\varphi$. We define $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu}y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$. The Feynman rules for this theory were already worked-out by Feynman [17, 18]. where we use his gauge, $\partial^\mu\bar{h}_{\nu\mu} = 0$.

Concerning the non-zero value of Λ, $\Lambda/\kappa^2 \sim (0.0024\ \mathrm{eV})^4$ [22], we see that it is so small on the EW scale represented by the Higgs mass that its main effect in our loop corrections will be to provide an IR regulator for the graviton infrared (IR) divergences. This subtle point should be understood as follows. Our non-zero value of Λ means that the true background metric is that of de Sitter, not that of Minkowski. We study the theory using the Minkowski background as an approximate representation of the actual de Sitter one, adding in the required corrections when we probe that regime of space-time where the correction is significant: this is in the far IR where the effective graviton IR regulator mass, already noted by

Feynman [18], represents the effect of the de Sitter curvature in our loop calculus. Thus, we are not in violation of the no-go theorems in Refs. [23, 24].

The main stumbling block of the Feynman formulation is already evident in Fig. 1, wherein we see that, by naive power counting, the graphs have superficial degree of divergence $D = 4$, so that, even if we take gauge invariance into account, we still have $D_{eff} \geq 0$, and higher loops give higher values of D_{eff}. The theory is thus, from this perspective, non-renormalizable as it is well-known.

As we explain in Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16], this bad UV behavior can be greatly improved by applying the methods of amplitude-based, exact resummation theory to arrive at what we have called resummed quantum gravity. We review this approach to the UV behavior of quantum gravity in the next section.

III. RESUMMED QUANTUM GRAVITY

The basic strategy we use is to make an exact re-arrangement of the Feynman formulated perturbative series for Einstein's theory with the idea that the interactions in the theory actually tame the attendant bad UV behavior dynamically. Intuitively, Newton's force is attractive between two positive masses, so that it becomes repulsive for negative mass-squared as we have in the deep Euclidean regime of the UV and this repulsion, in Feynman's overall space-time path-space approach, would lead to severe damping of UV propagation, thereby taming the otherwise bad UV behavior. This all would be consistent with Weinberg's asymptotic safety approach as recently developed in Refs. [2, 3, 4, 25, 26, 27]. As we have shown in Refs. [7], exact resummation of the IR dominated part of the proper self-energy function for a scalar particle of mass m gives the exact re-arrangement

$$i\Delta'_F(k)|_{\text{Resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \qquad (3)$$

where we have [7]

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2}$$

when the use the IR regulator mass λ for the graviton to represent the leading effect of the small recently discovered [22] cosmological constant, an effect Feynman already pointed-out in Ref. [18], for example. The residual self-energy function Σ'_s starts in $\mathcal{O}(\kappa^2)$, so we may drop it in calculating one-loop effects.

We note the following:

1. In the deep UV, explicit evaluation gives

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right), \qquad (4)$$

so that the resummed propagator falls faster than any power of $|k^2|$! Observe: in the Euclidean regime, $-|k^2| = k^2$ so there is trivially no analyticity issue here.

2. If m vanishes, using the usual $-\mu^2$ normalization point we get $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{\mu^2}{|k^2|}\right)$ which again vanishes faster than any power of $|k^2|$! This means that one-loop corrections are UV finite! Indeed, as we show in Ref. [7], all quantum gravity loops are UV finite!

3. In non-Abelian gauge theories, the Källén-Lehmann representation cannot be used to show that the attendant gauge field renormalization constant Z_3 is formally less than 1 so that Weinberg's argument [1] that the attendant spectral density condition, in an obvious notation, $\rho_{\text{K-L}}(\mu) \geq 0$ prevents the graviton propagator from falling faster than $1/k^2$ does not hold in such theories, as he has intimated himself.

4. One might think that Ward-Takahashi identities would require that the vertex correction resummation compensate any propagator resummation so that the net effect in a loop calculation if both vertices and propagators are resummed is to leave the power counting in the UV for the loop unchanged [28]. In fact, if we put the square root of the propagator as a factor for each leg entering or leaving a vertex and resum as well the corresponding large IR effects in the vertex, we still have exponential damping because the large resummed IR effects in the vertex behave subdominantly [29] in the deep UV and this does not cancel the propagator fall-off.

5. The fact that we find that the dynamics of quantum gravity leads to UV finiteness is consistent with both the asymptotic safety approach of Weinberg, as recently developed by Refs. [2, 3, 4, 25, 26, 27] and with the recent leg renormalizable result of Kreimer [30], wherein he finds at least for the pure gravity part of Einstein's theory, using the Hopf-algebraic Dyson-Schwinger equation realization of renormalization theory [31], that, while quantum gravity is non-renormalizable order by order in perturbation theory, there is an infinite set of relations among residues of the respective amplitudes so that when all are imposed only a finite number of unknown constants obtain, i.e., he finds in this way more evidence that quantum gravity is non-perturbatively renormalizable.

We have called our representation of the quantum theory of general relativity resummed quantum gravity (RQG). A number of applications have been worked-out in Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. We turn to its implications [32] for Planck scale cosmology in the next section.

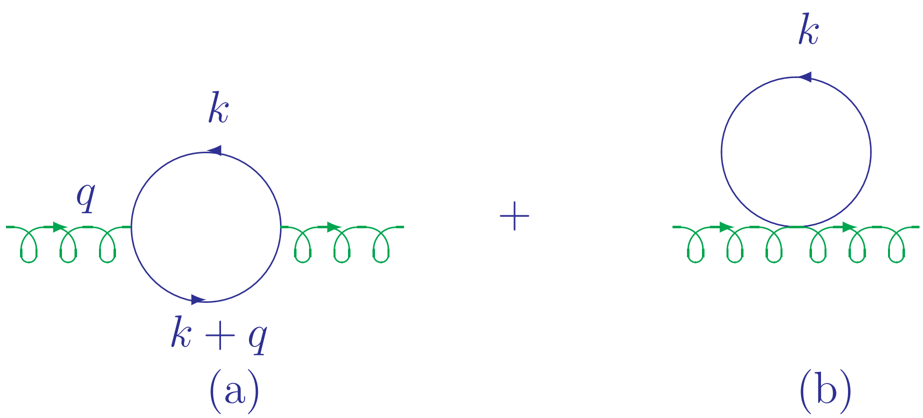

FIG. 1: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

IV. PLANCK SCALE COSMOLOGY

Consider the graviton propagator in the theory of gravity coupled to a massive scalar(Higgs) field [17, 18]. We have the graphs in Fig. 2 in addition to that in Fig. 1. Using the resummed theory, we get that the Newton potential becomes

$$\Phi_N(r) = -\frac{G_N M}{r}(1 - e^{-ar}), \tag{5}$$

for

$$a \cong 0.210 M_{Pl}, \tag{6}$$

so that we have

$$G(k) = G_N/(1 + \frac{k^2}{a^2})$$

, which implies fixed point behavior for $k^2 \to \infty$, in agreement with the asymptotic safety approach of Weinberg as recently developed in Refs. [2, 3, 4, 25, 26, 27]. Indeed, in Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16], we have shown that we are in agreement with the results in Refs. [2, 3, 4, 25, 26, 27] on several aspects of the UV limit of quantum gravity, such as the final state of Hawking radiation [33, 34] for an originally very massive black hole. Let us note for completeness that Ref. [35] gets a similar result in loop quantum gravity [36]. Here we show that we also agree with the Planck scale cosmology phenomenology developed in Refs. [2, 3]. We believe this strengthens the case for asymptotic safety.

Specifically, Bonanno and Reuter [2, 3] present a phenomenological approach to Planck scale cosmology wherein the starting point is the Einstein-Hilbert theory

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda)\,. \tag{7}$$

Using the phenomenological exact renormalization group for the Wilsonian coarse grained effective average action in field space, the authors in Refs. [2, 3, 25] show that attendant running Newton constant $G_N(k)$ and running cosmological constant $\Lambda(k)$ approach UV fixed points as k goes to infinity in the deep Euclidean regime – $k^2 G_N(k) \to g_*,\ \Lambda(k) \to \lambda_* k^2$ for $k \to \infty$ in the Euclidean regime. Due to the thinning of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Ref. [37] are obviated [38].

The contact with cosmology then proceeds as follows: invoking a phenomenological connection between the momentum scale k characterizing the coarseness of the Wilsonian graininess of the average effective action and the cosmological time t, the authors in Ref. [2, 3] show the standard cosmological equations admit the following extension:

$$(\frac{\dot{a}}{a})^2 + \frac{K}{a^2} = \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho \tag{8}$$

$$\dot{\rho} + 3(1+\omega)\frac{\dot{a}}{a}\rho = 0 \tag{9}$$

$$\dot{\Lambda} + 8\pi\rho\dot{G}_N = 0 \tag{10}$$

$$G_N(t) = G_N(k(t)) \tag{11}$$

$$\Lambda(t) = \Lambda(k(t)) \tag{12}$$

in a standard notation for the density ρ and scale factor $a(t)$ with the Robertson-Walker metric representation as

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{13}$$

where $K = 0, 1, -1$ corresponds respectively flat, spherical and pseudo-spherical 3-spaces for constant time t for a linear relation between the pressure p and ρ

$$p(t) = \omega\rho(t). \tag{14}$$

The functional relationship between the respective momentum scale k and the cosmological time t is de-

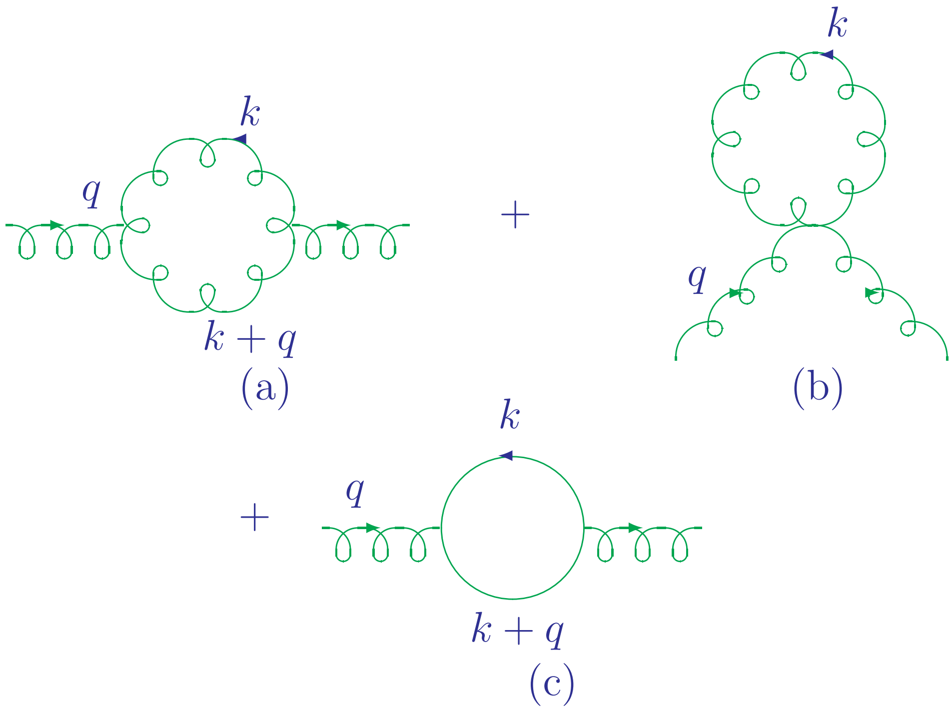

FIG. 2: The graviton((a),(b)) and its ghost((c)) one-loop contributions to the graviton propagator. q is the 4-momentum of the graviton.

termined phenomenologically via

$$k(t) = \frac{\xi}{t} \tag{15}$$

with the positive constant ξ determined phenomenologically .

Using the phenomenological, exact renormalization group (asymptotic safety) UV fixed points as discussed above for $k^2G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ the authors in Refs. [2, 3] show that the system in (12) admits, for $K = 0$, a solution in the Planck regime ($0 \le t \le t_{\rm class}$, with $t_{\rm class}$ a few times the Planck time t_{pl}), which joins smoothly onto a solution in the classical regime ($t > t_{\rm class}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.

The fixed-point results g_*, λ_* depend on the cut-offs used in the Wilsonian coarse-graining procedure. The key properties of g_*, λ_* used for the analysis in Refs. [2, 3](hereafter referred to as the B-R analysis) are that they are both positive and that the product $g_*\lambda_*$ is cut-off/threshold function independent. Here, we present the predictions for these UV limits as implied by resummed quantum gravity theory, providing a more rigorous basis for the B-R analysis.

Specifically, in addition to our UV fixed-point result for $G_N(k) \to a^2G_N/k^2 \equiv g_*/k^2$, we also get UV fixed point behavior for $\Lambda(k)$: using Einstein's equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu} \tag{16}$$

and the point-splitting definition

$$\begin{aligned}\varphi(0)\varphi(0) &= \lim_{\epsilon\to 0}\varphi(\epsilon)\varphi(0)\\ &= \lim_{\epsilon\to 0} T(\varphi(\epsilon)\varphi(0))\\ &= \lim_{\epsilon\to 0}\{:(\varphi(\epsilon)\varphi(0)): + <0|T(\varphi(\epsilon)\varphi(0))|0>\}\end{aligned} \tag{17}$$

we get for a scalar the contribution to Λ, in Euclidean representation,

$$\begin{aligned}\Lambda_s &= -8\pi G_N \frac{\int d^4k}{2(2\pi)^4}\frac{(2\vec{k}^2 + 2m^2)e^{-\lambda_c(k^2/(2m^2))\ln(k^2/m^2+1)}}{k^2+m^2}\\ &\cong -8\pi G_N[\frac{3}{G_N^2 64\rho^2}], \quad \rho = \ln\frac{1}{\lambda_c}\end{aligned} \tag{18}$$

with $\lambda_c = \frac{2m^2}{M_{Pl}^2}$. For a Dirac fermion, we get -4 times this contribution.

From these results, we get the Planck scale limit

$$\begin{aligned}&\Lambda(k) \to k^2\lambda_*,\\ &\lambda_* = \frac{1}{960\rho_{avg}}(\sum_j n_j)(\sum_j (-1)^{F_j} n_j)\end{aligned} \tag{19}$$

where F_j is the fermion number of j, n_j is the effective number of degrees of freedom of j, and ρ_{avg} is the average value of ρ – see Ref. [32].

All of the Planck scale cosmology results of Bonanno and Reuter [2, 3] hold, but with definite results for the limits $k^2G(k) = g_*$ and λ_* for $k^2 \to \infty$: solution of the horizon and flatness problem, scale free spectrum of primordial density fluctuations, initial entropy, etc., all provided by Planck scale quantum physics.

For reference, our UV fixed-point calculated here, $(g_*, \lambda_*) \cong (0.0442, 0.232)$, can be compared with the estimates of B-R, $(g_*, \lambda_*) \approx (0.27, 0.36)$, with the understanding that B-R analysis did not include SM matter action and that the attendant results have definitely cut-off function sensitivity. The qualitative results that g_* and λ_* are both positive and are significantly less than 1 in size with $\lambda_* > g_*$ are true of our results as well. We argue that this puts the results in Refs. [2, 3] on a more firm theoretical basis.

V. SUMMARY

In this discussion, we have shown that the application of exact amplitude-based resummation methods, where we stress that for the 1PI 2-point function for example we have resummed the IR part of its loops in Feynman's formulation of Einstein's theory for arbitrary values of the respective external line momenta, we achieve the first *first principles* calculations of the UV limits of the dimensionless gravitational and cosmological constants. We have shown that these results agree with those found by the phenomenological asymptotic safety based exact, Wilsonian field space renormalization group analysis of Refs. [2, 3, 4, 25, 26, 27] and that our results support the properties of these limits as they are used in Refs. [2, 3] to formulate Planck scale cosmology as an alternative to the standard inflationary cosmological paradigm of Guth and Linde [5, 6]. We believe our analysis puts the arguments in Refs. [2, 3] for such an alternative on a more firm theoretical basis. Ultimately, we do expect experiment to make a choice between the two.

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